



Exploring Space Through MATH

Applications in Precalculus

EDUCATOR
EDITION

Simulating Weightlessness

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will

- understand and write parametric equations that model the path of NASA's C-9 jet used to simulate microgravity;
- determine the distance and height of a projectile after t seconds;
- analyze given parameters to solve a real-life problem situation;
- find the maximum time and distance of one parabolic maneuver and interpret their significance; and
- use time as a parameter in parametric equations.

Prerequisites

Students should have experience using the TI-Nspire handheld to graph parametric equations involving projectile motion. Students should have prior knowledge of the properties of a parabolic function.

Background

This problem applies mathematical principles in NASA's human spaceflight.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. Through exploration we seek answers to fundamental questions about our existence, to discover and respond to new environments, to put in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown; we learn and we understand. As we take the first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

In our quest to explore, humans will have to adapt to functioning in a variety of gravitational environments. The Earth, Moon, Mars and space all have different gravitational characteristics. Earth's gravitational force is referred to as one Earth gravity, or 1 g . Since the Moon has less mass than the Earth, its gravitational force is only one-sixth that of Earth's, or 0.17 g . The gravitational force on Mars is equivalent to about 38% of Earth's gravity, or 0.38 g . The gravitational force in space is called microgravity and is very close to zero- g .

Key Concepts

Parametric equations,
parabolic functions

Problem Duration

70 minutes

Teacher Prep Time

15 minutes

Technology

Computer with Internet
access and projector,
TI-Nspire™ handhelds

Materials

- TI-Nspire Student Edition: *Precal-ST_Nspire_SimWt.pdf*
- TI-Nspire Student document: *Precal-ST_SimWt.tns*
- Video: *NASA Reduced Gravity Student Flight Opportunity*

Skills

Operations with
trigonometric functions,
translating angles

NCTM Principles

- Algebra
- Problem Solving
- Communication
- Connections
- Representations

Common Core Standards

- Modeling



When astronauts are in orbit, either in the space shuttle or on the International Space Station, they are still affected by Earth's gravitational force. However, astronauts maintain a feeling of weightlessness, since both the vehicle and crewmembers are in a constant state of free-fall. Even though they are falling towards the Earth, they are traveling fast enough around the Earth to stay in orbit. During orbit, the gravitational force on the astronauts relative to the vehicle is close to zero-g.



Figure 1: C-9 jet going into a parabolic maneuver



Figure 2: Astronaut crew training for an upcoming mission on the C-9 jet

The C-9 jet is one of the tools utilized by NASA to simulate the gravity, or reduced gravity, that astronauts feel once they leave Earth (Figure 1). The C-9 jet flies a special parabolic pattern that creates several brief periods of reduced gravity. A typical NASA C-9 flight leaves from Houston, Texas, and goes out over the Gulf of Mexico. It lasts about two hours and completes between 40 and 60 parabolas. These reduced gravity flights are performed so that astronauts, as well as researchers and their experiments, can experience the simulated gravitational forces of the Moon and Mars and the microgravity of space.

By using the C-9 jet as a reduced gravity research laboratory, astronauts can simulate different stages of spaceflight; allowing crewmembers to practice what might occur during a real mission. These reduced gravity flights provide the capability for the development and verification of space hardware, scientific experiments, and other types of research (Figure 2). NASA scientists can also use these flights for crew training, including exercising in reduced gravity, administering medical care, performing experiments, and many other aspects of spaceflight that will be necessary for an exploration mission.

NCTM Principles and Standards

Algebra

- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases.
- Use symbolic algebra to represent and explain mathematical relationships.

**Problem Solving**

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.

Common Core Standards

- Modeling

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (20 minutes)

- Have students read the background section aloud to the class.
- Play the video, *NASA Reduced Gravity Student Flight Opportunity* (13:35 min.) at: http://www.nasa.gov/multimedia/videogallery/index.html?media_id=65696581
- Discuss the background and the video with the class.

2 – Explore (15 minutes)

- Place students in groups of 3–4.
- Distribute the TI-Nspire™ document, *Precal- ST_SimWt.tns*.
- Have students read the problem situation on page 1.7.
- Allow students to work together on questions 1.8–1.12.
- Optional: To save time, have students write answers to questions 1.9–1.12 on the Student Edition rather than typing answers on the handheld.

3 – Explain (15 minutes)

- With students still in groups, have them answer questions 1.13–1.17.
- Move about the class to assist students as needed.
- Encourage student discussion within groups.

**4 – Extend** (10 minutes)

- With students still in groups, have them answer questions 1.18–1.19.
- Move about the class to assist students as needed.
- Encourage student discussion within groups.

5 – Evaluate (10 minutes)

- With students still in groups, have them answer questions 2.0–2.3.
- Allow each group to discuss their answers with the entire class.

Simulating Weightlessness**Solution Key**

Note to teacher: Instruct students to open TI-Nspire document, Precal-ST_SimWt.tns on their handhelds. You may choose to have students record their work directly in their handhelds or on their worksheets in the provided spaces. A solution key is provided below, as well as in the educator's version of the TI-Nspire document, Precal-ED_SimWt.tns.

Directions: On the TI-Nspire™ handheld, open the document, Precal-ST_SimWt. Read through the problem set-up (pages 1.2–1.8) and complete the embedded questions.

Problem

To prepare for an upcoming mission, an astronaut participates in a C-9 flight simulating microgravity. The pilot flies out over the Gulf of Mexico, dives down to increase to a maximum speed, then climbs up until the nose of the C-9 is at a 45° angle with the ground. At this point, the velocity of the jet is 123.333 meters per second (about 283 mph) and the altitude is 9,144 meters (about 30,000 ft). To go into a parabolic maneuver, the pilot cuts the thrust of the engine, letting the nose of the jet rise, and then comes back down at a -45° angle with the ground. Ending the parabolic maneuver, the pilot throttles the engine back up and begins another dive to prepare for the next parabola. The pilot is able to complete 50 parabolas during the 2-hour flight.

Figure 3 shows the movement of the jet during a typical flight. The parabolic maneuver, where microgravity is felt, is highlighted.

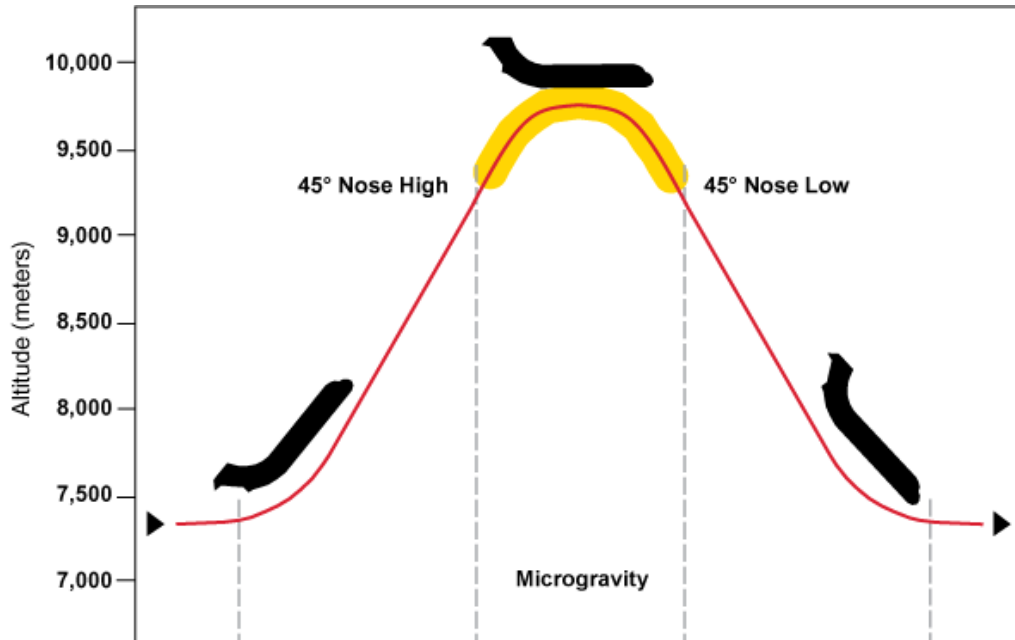


Figure 3: A typical microgravity maneuver

Parametric equations describing projectile motion are of the form:

$$x_{1t} = v_0 \cos(\theta)t$$

$$y_{1t} = -\frac{1}{2}gt^2 + v_0 \sin(\theta)t + h_0$$

The variable, g , is the acceleration due to gravity; v_0 is the initial velocity; t is time; θ is the angle of ascent; and h_0 is the initial altitude of the C-9 jet when it enters the parabolic maneuver.

On Earth, acceleration due to gravity is approximately 9.8 m/s^2 . For this problem, other environmental influences (such as air resistance) should be disregarded.

Directions: Answer questions 1.9–2.3 in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth, and label with the appropriate units.

The graph on page 1.8 represents the graph of a C-9 parabolic maneuver as previously defined. Grab the point that lies on the curve and observe the direction and magnitude of the arrows (velocity vectors) as you drag the point along the flight path. Write your answers to the questions in the space provided below.

- 1.9 What is happening to the angle of the jet's ascent/descent as it moves along its parabolic path?

The angle of the jet's ascent decreases until the jet reaches the top of the parabola where the angle is zero. The jet then descends and the angle of descent, which is negative, increases in magnitude.



- 1.10 What is happening to the vertical velocity vector as the jet moves along its parabolic path?

The vertical velocity vector, which has a positive direction as the plane ascends, decreases in magnitude to zero at the top of the parabola. It then increases in magnitude as the jet descends, but has negative direction.

- 1.11 What is happening to the horizontal velocity vector as the jet moves along its parabolic path?

The horizontal velocity vector changes as the jet's angle changes. It gets closer to zero as the jet reaches the maximum height of the parabolic maneuver.

- 1.12 Explain the reasons for the changes in the vertical and horizontal velocity vectors.

The vertical velocity is subject to the pull of gravity which makes its value decrease, while the horizontal velocity has no forces acting on it (since air resistance is not considered).

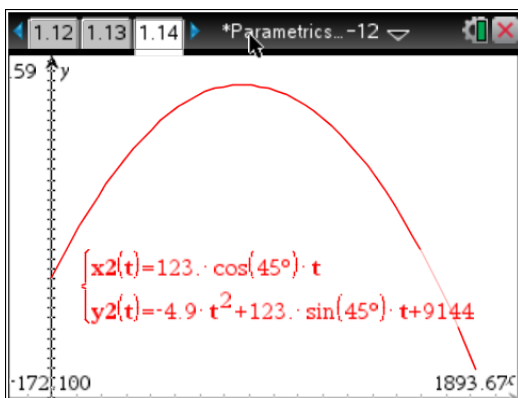
- 1.13 Determine the parametric equations to model the problem situation and graph them on page 1.14.

Teacher Note: To graph parametric equations, go to Menu>Graph Entry/Edit>Parametric.

Parametric equations:

$$x_{1t} = 123.333 \cos(45^\circ) t$$

$$y_{1t} = -4.9t^2 + 123.333 \sin(45^\circ) t + 9144$$



- 1.15 How many seconds does it take the C-9 jet to reach its maximum height?

$$t = \frac{-b}{2a}$$

$$t = \frac{-123.333 \sin(45^\circ)}{-9.8}$$

$$t = 8.899 \text{ sec}$$



- 1.16 What is the maximum time the C-9 is in one parabolic maneuver experiencing microgravity?

The total time would be the time it takes to reach its maximum height multiplied by 2.

$t = 17.798$ seconds

- 1.17 Use your answer from the previous question to calculate the total horizontal distance (in meters) that the jet has traveled during the maneuver.

$$x_{1t} = v_0 \cos(\theta)t$$

$$x_{1t} = 123.333 \cos(45^\circ)(17.798)$$

$$x_{1t} = 1552.160 \text{ meters}$$

- 1.18 What is the maximum height that the C-9 jet reaches?

$$y_{1t} = -4.9(8.899)^2 + 123.333 \sin(45^\circ)(8.899) + 9144$$

$$y_{1t} = 9532.040 \text{ meters}$$

- 1.19 What is the height and horizontal distance traveled by the C-9 after 5 seconds?

Substitute 5 seconds in for time in both parametric equations.

Height at 5 seconds:

$$y_{1t} = -4.9t^2 + 123.333 \sin(45^\circ)t + 9144$$

$$y_{1t} = -4.9(5 \text{ sec})^2 + 123.333 \sin(45^\circ)(5 \text{ sec}) + 9144$$

$$y_{1t} = 9457.550 \text{ meters}$$

Horizontal distance traveled at 5 seconds:

$$x_{1t} = 123.333 \cos(45^\circ)t$$

$$x_{1t} = 123.333 \cos(45^\circ)(5 \text{ sec})$$

$$x_{1t} = 436.048 \text{ meters}$$



1.20 What is the height and distance traveled by the C-9 after 15 seconds?

Substitute the time of 15 seconds in both parametric equations.

Height at 15 seconds:

$$y_{1t} = -4.9t^2 + 123.333 \sin(45^\circ)t + 9144$$

$$y_{1t} = -4.9(15 \text{ sec})^2 + 123.333 \sin(45^\circ)(15 \text{ sec}) + 9144$$

$$y_{1t} = 9349.610 \text{ meters}$$

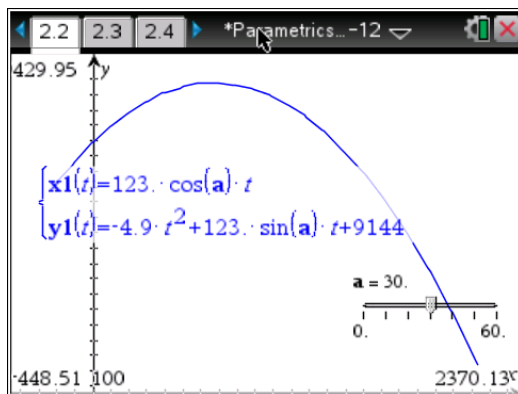
Horizontal distance traveled at 15 seconds:

$$x_{1t} = 123.333 \cos(45^\circ)t$$

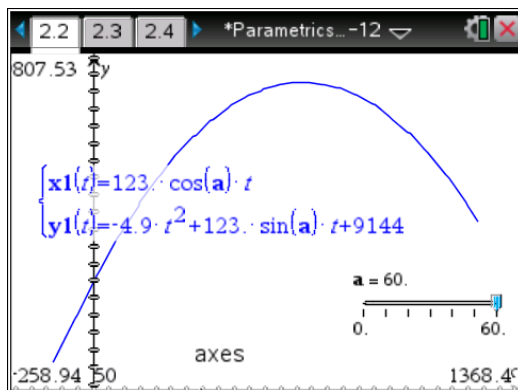
$$x_{1t} = 123.333 \cos(45^\circ)(15 \text{ sec})$$

$$x_{1t} = 1308.140 \text{ meters}$$

2.1 Explore how the parabolic maneuver changes by keeping the same initial velocity, but changing the jet's angle of ascent to 30° and 60° by using the graph and slider on page 2.2. How does each angle affect the parabolic maneuver?



For an angle of ascent of 30° , the parabola spans less horizontal distance than a 45° angle.



For an angle of ascent of 60° , the parabola spans less horizontal distance than the 45° angle.



- 2.3 Now that you have explored several angles of ascent, calculate the time spent in microgravity for the 60° angle explored on page 2.2. Why do you think the mission planner chose 45° as the jet's angle of ascent?

$$t = \frac{-123.333 \sin(60^\circ)}{-9.8}$$

$$t = 10.898 \text{ sec}$$

$$t_{\text{total}} = 2(10.898)$$

$$t_{\text{total}} = 21.798 \text{ sec}$$

Although 60° gives more time in a parabolic maneuver, the 45° angle is safer for both the crew and experiments.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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